# Dynamics of Helical Vortex Systems

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**ABSTRACT**
Fundamental mechanisms at play in the dynamics of the two helical-vortex system are highlighted by DNS associated to theory: merging mechanisms at large, medium or small pitch, leapfrogging/overtaking dynamics before merging at low pitch.

## 1. Introduction

Blade-tip vortices generated in the wake of rotating devices often have – at least locally – a *helical symmetry*, i.e. the flow is invariant through combined translation and rotation along their common axis. Examples are the wake of wind turbines, marine propellers... Helical symmetry has been used for a long time now to derive wake models based on polygons of inviscid vortex filaments. It is then possible to express induced velocity fields, translation or rotation speeds of the structures themselves, and to investigate their instabilities. Although very fruitful, such approaches naturally preclude the study of intrinsic viscous dynamics, such as helical vortex spreading, merging of several vortices...

Similarly to the case of plane or axisymmetric flows, helically symmetric flows can be described using only two space variables, allow the use of a streamfunction, and their dynamics is governed by a set of two coupled equations for the helical vorticity and velocity components. Following this line, a direct numerical simulation (DNS) code named HELIX [1] has been devised to compute the time evolution of the flow within the framework of helical symmetry. The code takes into account three-dimensional effects but is based on two-dimensional numerical integration techniques, allowing for long integration times and high Reynolds numbers. Helically symmetric flows and their DNS are briefly described in § 2.

The code has been first used to investigate the merging of two helical vortices. Several scenarios have been brought out depending on the pitch, as presented in § 3. Helical vortices with a long pitch behave much alike straight vortices. Below a critical pitch however, instability sets in, causing the mechanism of merging to change drastically. For small-pitch vortices, the principles of a theoretical study of the dynamics before merging is presented in § 4. Helical vortices with small pitch are shown to bear many similarities with coaxial vortex ring systems, they undergo the well-known leapfrogging dynamics, but other dynamics have also been predicted.

## 2. Flows with Helical Symmetry

Flows with a helical symmetry of pitch $h = 2\pi L$ ($L$ is the *reduced pitch*) have their velocity, pressure and vorticity fields invariant under the combination of an axial translation of $\Delta z$ and a rotation of angle $\Delta z/L$ about the same axis. They can be described using only two space variables, namely radius $r$ and variable $\varphi = \theta - z/L$, where $(r, \theta, z)$ denote the usual polar coordinates. Under the assumption of helical symmetry, incompressibility implies that the velocity field can be expressed using only two functions as $\vec{u} = u_B \vec{e}_B + \alpha(r) \boldsymbol{\nabla}\psi \times \vec{e}_B$ where $u_B$ designates the velocity component along the unit vector $\vec{e}_B$ tangent to helical lines, $\psi(r, \varphi, t)$ the helical streamfunction and $\alpha(r) = (1 + r^2/L^2)^{-1/2}$ a geometrical factor. The governing equations can be written as a set of two dynamical equations for $u_B$ and $\omega_B \equiv \vec{\omega} \cdot \vec{e}_B$:

$$\begin{aligned} \partial_t u_B + NL_u &= VT_u \\ \partial_t \omega_B + NL_\omega &= VT_\omega \,. \end{aligned} \qquad (1)$$

The expressions of nonlinear (NL) and viscous terms (VT) can be found in [1]. The HELIX code implements the time integration of (1) over a circular domain. Nonsingular boundary conditions are enforced at the axis and potential flow at the outer boundary. Fourier series for the $2\pi$-periodic variable $\varphi$, and 2nd order finite differences along $r$. The time advance is performed using second order backward discretisation of the temporal derivative. Nonlinear terms appear explicitly through 2nd order Adams–Bashforth extrapolation whereas the viscous terms are integrally treated implicitly.

In the following, quantities are made dimensionless using the initial helix radius $R_0$ as length scale and $R_0^2/\Gamma$ as time scale, $\Gamma$ being the circulation of each of the two helical vortices considered. The Reynolds number is defined as $Re = \Gamma/\nu$. Details on how initial conditions are elaborated in this context are available in [2].

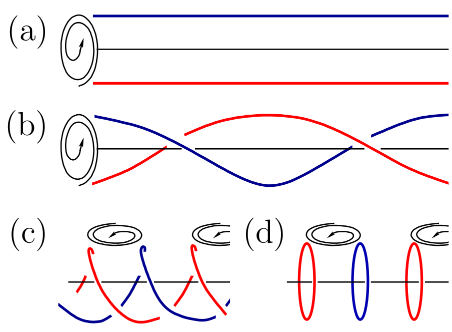


Fig. 1 Merging of vortices: (a) straight vortices, (b) large-pitch helical vortices, (c) small-pitch helical vortices, (d) array of vortex rings. The arrows qualitatively indicate the merging dynamics.

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## 3. Merging of Two Helical Vortices: Effect of the Pitch $L$

Using the DNS code, we investigated the various mechanisms through which two helical vortices merge. A set of simulations were performed for pitches ranging from $L = 0.5$ to $L = 3$ and Reynolds numbers from $Re = 1\ 000$ to $10\ 000$. The initial core size was chosen relatively large at $a_0 = 0.2$ in order to shorten the phases preceding merging itself. At large pitch ($L \geq 1.9$), helical vortex merging is qualitatively analogous to the merging of 2D vortices (figure 1a), and the process can be described as a *convective merging at axis* (figure 1b). On the opposite side, helical vortices at small pitch ($L \leq 1.2$) are potentially prone to an inviscid instability [3] whereby facing coils of the two vortices move towards each other (figure 1c). This *merging induced by instability* is found to be, for helical vortex coils, a process analogous to the pairing instability of an array of vortex rings (figure 1d). At intermediate pitch ($1.2 < L < 1.9$), the merging is purely viscous, and can result either in a single vortex along the axis, in a single helical vortex, or in a cylindrical vorticity layer. The different types of merging are summarized in figure 2 in the $(Re, L)$ plane, while extensive details on physical mechanisms can be found in [2].

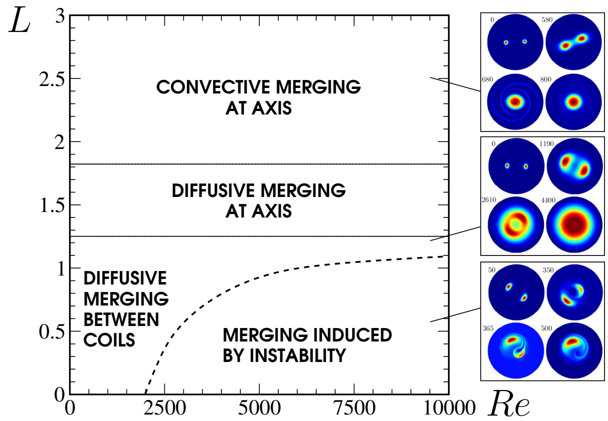


Fig. 2 Merging of two helical vortices: different scenarios in the $(Re, L)$ plane. Right: typical time evolutions of $\omega_B$ is a plane orthogonal to the helix axis.

## 4. Leapfrogging Dynamics of Two Small Pitch Helical Vortices

When the initial core size $a_0$ is taken sufficiently small (0.05 for instance), merging does not occur as soon as presented in § 3. At small pitch, the two vortices destabilize through linear instability, and a rich nonlinear interaction dynamics sets in before merging, which consists of a sequence of elementary events with one of the vortices passing "through" the other; note that the term "through" would be appropriate for rings, it designates here the counterpart for helical vortices, as illustrated in figure 3a. One major finding is that vortices behave differently depending on the way they are being disturbed initially. If displaced azimuthally (or axially) towards each other, they undergo a succession of *leapfrogging events*: vortex 1 passes through vortex 2, then vortex 2 through vortex 1 and so on until they merge. If displaced radially however, they first undergo a succession of *overtaking events*: vortex 1 passes through vortex 2 several times before the alternating leapfrog begins, eventually leading to merging.

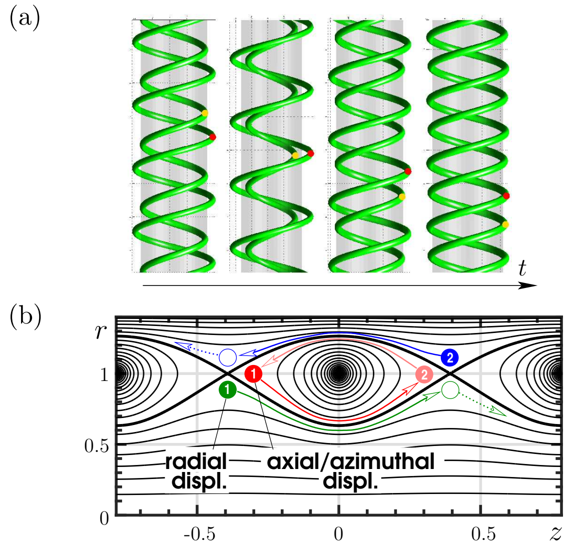


Fig. 3 (a) Elementary event in the nonlinear dynamics of two helical vortices. (b) Iso-Hamiltonian lines in the $(r, z)$ plane, showing several types of vortex trajectories.

These dynamics can be accounted for theoretically at small pitch by replacing helical vortices by two infinite arrays of vortex rings (figures 1c-d) and, if viscosity is left aside, adopting a Hamiltonian formulation. Conservation laws then lead the vortices to follow specific lines in the $(r, z)$ plane, as displayed in figure 3b. These results are confirmed by Biot-Savart simulations of inviscid helical vortices within the cut-off theory. In the viscous framework, DNS indicate that there is a transition from overtaking to leapfrog, and thereafter from leapfrog to merging, monitored by the ratio $a(t)/L$ [4].

## 5. Conclusion

Enforcing helical symmetry into a DNS code leads to highlight some fundamental mechanisms at play in systems of helical vortices, namely various scenarios of merging and of nonlinear dynamics following the instability at small pitch. This work was done in collaboration with Maurice Rossi, Benjamin Piton and Can Selçuk. HPC resources from GENCI-IDRIS (Grant No. 2018-2a1386) are acknowledged.

## References

[1] I. Delbende, M. Rossi, O. Daube, *Theo. Comp. Fluid Dyn.* 26 (2012), 148–175.

[2] I. Delbende, B. Piton, M. Rossi, *Eur. J. Mech. B-Fluid* 49 (2015), 363–372.

[3] V.L. Okulov, J.N. Sørensen, *J. Fluid Mech.* 576 (2007), 1–25.

[4] C. Selçuk, I. Delbende, M. Rossi, *Fluid Dyn. Res.* 50(1) (2018), 011411.